Earth tides and Lense-Thirring effect

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Abstract

The general relativistic Lense-Thirring effect can be measured by inspecting a suitable combination of the orbital residuals of the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II. The solid and ocean Earth tides affect the recovery of the parameter by means of which the gravitomagnetic force is accounted for in the combined residuals. Thus an extensive analysis of the perturbations induced on these orbital elements by the solid and ocean Earth tides is carried out. It involves the $l = 2$ terms for the solid tides and the $l = 2, 3, 4$ terms for the ocean tides. The perigee of LAGEOS II turns out to be very sensitive to the $l = 3$ part of the ocean tidal spectrum, contrary to the nodes of LAGEOS and LAGEOS II. The uncertainty in the solid tidal perturbations, mainly due to the Love number k_2, ranges from 0.4% to 1.5%, while the ocean tides are uncertain at $5\% - 15\%$ level. The obtained results are used in order to check in a preliminary way which tidal constituents the Lense-Thirring shift is sensitive to. In particular it is tested if the semisecular 18.6-year zonal tide really does not affect the combined residuals. It turns out that, if modeled at the level of accuracy worked out in the paper, the $l = 2, 4\ m = 0$ and also, to a lesser extent, the $l = 3\ m = 0$ tidal perturbations cancel out .

1   Introduction

If the Earth was perfectly spherical, according to Newton its gravitational potential would be $GM_\oplus/r$, where G is the Newtonian gravitational constant and $M_\oplus$ is the mass of the Earth; the path of any artificial satellite in orbit around it would be a Keplerian ellipse with the Earth in one of its foci. This ellipse would remain fixed in inertial space and neither its shape nor its size would change; it could be parameterized by means of the so called Keplerian orbital elements [*Sterne*, 1960; *Allison and Ashby*, 1993]. In reality, the behavior of an object orbiting the Earth is much more complicated because our planet is not perfectly spherical due to several factors. First of all, the centrifugal force caused by its diurnal rotation makes the Earth oblate; its gravitational field, called geopotential, is no longer central but must be developed in a multipolar harmonic expansion [*Kaula*, 1966]:

$$U = \frac{GM_\oplus}{r}\{1 + \sum_{l=2}^{\infty}\sum_{m=0}^{l}(\frac{R_\oplus}{r})^l P_{lm}(\sin\phi)[C_{lm}\cos m\lambda + S_{lm}\sin m\lambda]\} \tag{1}$$

where $R_\oplus$ is the Earth's mean equatorial radius, P_{lm} is the associated Legendre function of degree l and order m, and ϕ and λ are terrestrial latitude and east Greenwich longitude. C_{lm} and S_{lm} are the adimensional Stokes or geopotential coefficients [*Lemoine et al.,* 1998] . Second, the shape of the Earth changes also in time due to the action of the solid, ocean and atmospheric tides which periodically lift the surface, tilt the vertical and redistribute the masses on our planet. This effect can be accounted for in a similar fashion to the static, centrifugal part of the geopotential given by eq.(1) introducing in it time varying-coefficients [*Eanes et al.,* 1983]. The resulting effect of the static and dynamical non-sphericity of the Earth is that the path of any artificial satellite is more or less modified with respect to the unperturbed Keplerian ellipse. It is possible to account for these changes by assuming that the perturbed orbit can be still parameterized in terms of Keplerian elements which nevertheless slowly vary in time. This is so because the leading or point-mass term $U_0 = GM_\oplus/r$ is predominant over the perturbing terms [*Casotto*, 1989]. Obviously, there is also a wide class of non gravitational effects [*Milani et al.,* 1987] which contribute to modify the satellites' orbits; the concept of osculating ellipse fits also these cases and, in general, any situation in which a physical acceleration, small with respect to the Newtonian monopole, acts on the test body.

It should be pointed out that the experimental accuracy of techniques such as the Satellite Laser Ranging (SLR) has astonishingly grown up in the last decade: for example, today it is possible to measure the amplitude of a periodical perturbation on the node Ω of LAGEOS with an accuracy of some milliarcseconds (mas in the following). The accuracy for the same kind of measurement on the perigee ω is slightly worse (F. Vespe, private communication, 1999). This forces the researcher who wants to investigate some particular physical effect by means of near-Earth's satellites to be very careful in accounting for every tiny perturbation, of gravitational and non gravitational origin, which would be inevitably present in the record aliasing the recovered values of the quantities with which the phenomenon of interest is parameterized.

This is the case for the Lense-Thirring drag of inertial frames [*Lense and Thirring*, 1918; *Ciufolini and Wheeler*, 1995]. It is a feature of the gravitational field generated by every rotating mass predicted by general relativity which consists in a tiny, secular precession affecting the node Ω and the perigee ω of the orbit of a not too far freely falling test body. For LAGEOS and LAGEOS II SLR satellites the gravitomagneic precessional rates in the field of the Earth amount to:

$$\dot{\Omega}_{LT}^{LAGEOS} \simeq 31 \ mas/y, \tag{2}$$

$$\dot{\Omega}_{LT}^{LAGEOSII} \simeq 31.5 \ mas/y, \tag{3}$$

$$\dot{\omega}_{LT}^{LAGEOSII} \simeq -57 \ mas/y. \tag{4}$$

The major source of uncertainty in the detection of the gravitomagnetic shift turns out to be the mismodeling of the first two even zonal terms of the secular classical precessions caused by the oblateness of the Earth. *Ciufolini* [1996] has proposed a strategy which could allow to detect the Lense-Thirring effect at 20 % precision level by using as observable a suitable combination of the orbital residuals of LAGEOS and LAGEOS II which would allow to cancel out all the static and dynamical contributions of the first two even zonal terms of the geopotential along with the associated errors.

Among the various sources of systematical errors which may affect the recovery of the Lense-Thirring effect the Earth tides [*Cartwright*, 2000] play a relevant role.

The calculations presented here, in the context of the gravitomagnetic experiment, can be viewed as a preliminary sensitivity test of the Lense-Thirring shift to the tidal perturbations.

Moreover, in view of the wide application of the two LAGEOS in many fields of space science they could turn out to be useful for other experiments.

An evaluation from first principles of the amplitudes, the periods and the initial phases of the tidal perturbations on the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II is useful because:

• It allows to point out which tidal constituents the Lense-Thirring shift is mainly sensitive to, so that people can focus the researches on them

• It allows to get insight in how to update the orbit determination softwares. In this way the impact on the time series of those tidal constituents which will turn out to be more effective could be reduced along with the number of the parameters to be included in the least-squares solution

• For a given observational time span T_{obs} it allows to find those constituents whose periods are longer than it and consequently may act like superimposed bias (e.g. the 18.6-year and 9.3-year zonal tides and the K_1 $l = 3$ $p = 1$ $q = -1$ ocean diurnal tide on the perigee of LAGEOS II) corrupting the detection of the gravitomagnetic shift. In these cases, if we know their periods and initial phases we could try to fit and remove them from the time series[1] without affecting also the trend of interest or, at least, it should be possible to assess the mismodeling level induced by them on the detection of the Lense-Thirring trend so to include also these estimates in the final error budget

• It can be used as starting point for numerical simulations of the time series in order to check, e. g., the impact of the diurnal (m=1) and semidiurnal (m=2) tidal perturbations (not canceled out by the Ciufolini' s observable), as done in a further paper [*Pavlis and Iorio*, 2001].

Regarding the claimed cancellation of the perturbations induced by the first two even zonal static and dynamical terms of the geopotential by the proposed combined residuals, it has been checked, as far as the tides are concerned, if it really occurs and in this case at which level of accuracy it takes place, and if this feature can be extended to some other tidal constituents by updating and extending the calculations performed with the nominal amplitudes in [*Ciufolini et al.*, 1997 pag. 2714 eq.(20), eq.(21)]

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[1]If the period of the disturbing harmonic, assumed to be known, is shorter than the time series length the perturbation can be reliably viewed as an empirically fit quantity. But if its period is longer than the time series length it is not possible to fit and remove the harmonic without removing also the true linear trend of interest.

The paper is organized as follows: In Sec. 2 the perturbative amplitudes on Ω and ω of LAGEOS and LAGEOS II due to the solid tides are worked out. The calculation account for the frequency-dependence of Love numbers k_2 and the anelasticity of the Earth's mantle. For some selected tidal lines -the most effective in perturbing the satellites' orbits- also the effect of Earth's rotation and ellipticity, accounted for by the coefficients k_2^+, are investigated. In Sec. 3 analogous calculations for the ocean tidal spectrum are performed. In Sec. 4 the mismodeled perturbative amplitudes of the solid and ocean tidal spectrum are compared to the gravitomagnetic shift over 4 years in order to check which tidal constituents, over such temporal span, are mismodeled at 1% level of the Lense-Thirring effect. Subsequently, the obtained results are employed to check extensively the feasibility of the cancellation of the zonal tidal constituents by means of the formula proposed by Ciufolini. Sec. 5 is devoted to the conclusions.

The data employed in the present analysis are in the following table in which P is the orbital period and $P[X]$ is the period of the Keplerian element X:

$$a_{LAGEOS} = 12,270,km \tag{5}$$

$$a_{LAGEOSII} = 12,163,km \tag{6}$$

$$e_{LAGEOS} = 0.0045 \tag{7}$$

$$e_{LAGEOSII} = 0.014 \tag{8}$$

$$i_{LAGEOS} = 110,deg \tag{9}$$

$$i_{LAGEOSII} = 52.65,deg \tag{10}$$

$$P_{LAGEOS} = 0.1566,days \tag{11}$$

$$P_{LAGEOSII} = 0.1545,days \tag{12}$$

$$P[\Omega]_{LAGEOS} = 1,043.67,days \tag{13}$$

$$P[\Omega]_{LAGEOSII} = -569.21,days \tag{14}$$

$$P[\omega]_{LAGEOS} = -1,707.62,days \tag{15}$$

$$P[\omega]_{LAGEOSII} = 821.79,days \tag{16}$$

$$\tag{17}$$

2 Effect of the solid Earth tides on the nodal and apsidal lines of LAGEOS and LAGEOS II

The free space potential [*Dahlen,* 1972; *Smith,* 1974; *Wahr*, 1981b; *Melchior,* 1983; *Wang,* 1997] by means of which the Earth[2] responds to the tide generating potential $\Phi(\mathbf{r})$ [*Doodson,* 1921; *Cartwright and Tayler,* 1971; *Cartwright and Edden,* 1973; *Buellsfeld,* 1985; *Tamura,* 1987; *Xi,* 1987 ; *Hartmann and Wenzel,* 1995; *Roosbeek,* 1996], at $r = a$ for a near circular orbit satellite, is given by:

$$\phi = \frac{GM_{\oplus}}{R_{\oplus}} \sum_{l=2}^{\infty} \sum_{m=0}^{l} \frac{1}{a} (\frac{R_{\oplus}}{a})^l \sum_{p=0}^{l} \sum_{q=-\infty}^{+\infty} F_{lmp}(i) G_{lpq}(e) S_{lmpq}(\omega, \ \Omega, \mathcal{M}, \ \theta). \tag{18}$$

In eq.(18) $F_{lmp}(i)$m and $G_{lpq}(e)$ are the inclination and eccentricity functions respectively [*Kaula,* 1966] and the S_{lmpq} is given by:

$$S_{lmpq}(\omega, \ \Omega, \mathcal{M}, \ \theta) = A_{lm} \sum_f H_l^m k_{lm}^{(0)}(f) \cos(\sigma t + \psi_{lmpq} - \delta_{lmf}). \tag{19}$$

In eq.(19) ω, Ω and $\mathcal{M}$ are the argument of perigee, the longitude of ascending node and the mean anomaly, respectively, of the satellite, θ is the Greenwich sidereal time, while A_{lm} is given by:

$$A_{lm} = \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}}. \tag{20}$$

In eq.(20) the Condon-Shortley phase factor $(-1)^m$ has been neglected in order to compare the results of this Section to those by [*Bertotti and Carpino,* 1989]. The phase of the generic harmonic in eq.(19) is constituted by three terms. The first one, σt, is for every constituent of the tide generating potential a linear combination of the Mean Lunar Time τ and the mean ecliptical longitudes of the astronomical motions of the Moon and the Sun [*Dronkers ,* 1964] so that:

$$\sigma \equiv 2\pi f = j_1 \dot{\tau} + j_2 \dot{s} + j_3 \dot{h} + j_4 \dot{p} + j_5 \dot{N}' + j_6 \dot{p}_s. \tag{21}$$

Since $\tau = \theta - s$ eq.(21) becomes:

$$\sigma = j_1 \dot{\theta} + (j_2 - j_1)\dot{s} + j_3 \dot{h} + j_4 \dot{p} + j_5 \dot{N}' + j_6 \dot{p}_s. \tag{22}$$

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[2]It is assumed that the Earth is endowed with a solid inner core, a fluid outer core, and a solid mantle capped by a thin continental crust, without oceans and atmosphere. Substantially, the Earth is thought as a set of coupled harmonic oscillators showing a variety of resonant frequencies -the normal modes- which can be excited by the external forcing constituents of the tide generating potential [*Wahr,* 1981a].

The coefficients j_k, $k = 1, ..., 6$ are small integers which can assume negative, positive or null values. They are arranged in the so called Doodson number:

$$j_1(j_2 + 5)(j_3 + 5).(j_4 + 5)(j_5 + 5)(j_6 + 5), \tag{23}$$

by means of which every tidal constituent is named. In it the integer j_1 classifies the tides in long period or zonal ($j_1 = 0$), diurnal or tesseral ($j_1 = 1$) and semidiurnal or sectorial ($j_1 = 2$). According to the Doodson notation, the sum $\sum_f$ over the tidal constituents in eq.(19) stands for the sum over the integers $j_1, ...j_6$.

The second quantity appearing in the phase of eq.(19) is:

$$\psi_{lmpq} = (l - 2p)\omega + (l - 2p + q)\mathcal{M} + m(\Omega - \theta). \tag{24}$$

The parameter:

$$k_{lm}^{(0)}(f) = \sqrt{k_{lm}^R(f)^2 + k_{lm}^I(f)^2}, \tag{25}$$

is the modulus of the Love number k [*Love*, 1926; *Mathews et al.*, 1995] for a static, spherical Earth. In general, it is defined as the ratio of the free space gravitational potential $\phi(\mathbf{r})$, evaluated at the Earth' s equator, and the tide generating potential $\Phi(\mathbf{r})$ calculated at the mean equatorial radius. If the rotation and nonsphericity of the Earth are accounted for, there are different values of k for any l, m and f. In general, they are evaluated by convolving, in the frequency domain, the tide generating potential with the transfer function for a non rigid Earth. There are several theoretical calculations for the Love numbers k [*Wahr*, 1981b; *Wang*, 1994; *Mathews et al.*, 1995; *McCarthy*, 1996; *Dehant et al.*, 1999]. They differ in the choice of the Earth' s interior models [*Gilbert and Dziewonski*, 1975; *Dziewonski and Anderson*, 1981] adopted for the calculation of the transfer function and the departure from elasticity of the mantle' s behavior. Particularly important is also if they account for, or not, the normal modes in the diurnal band. In eq.(25) the IERS conventions [*McCarthy*, 1996] have been used for $k_{lm}^{(0)}(f)$:

$$k_{lm}^I(f) = Im\ k_{lm} + \delta k_f^{anel}, \tag{26}$$

$$k_{lm}^R(f) = Re\ k_{lm} + \delta k_f^{el}, \tag{27}$$

where $Im\ k_{lm}$ and $Re\ k_{lm}$ are the frequency-independent parts of Love numbers, and δk_f^{anel}, δk_f^{el} are the frequency-dependent corrections. They are important in the diurnal band, through δk_f^{el}, and in the zonal band with δk_f^{anel} due to the anelasticity of the mantle.

The factor δ_{lmf} is the phase lag of the response of the solid Earth with respect to the tidal constituent of degree l, order m and circular frequency σ induced by the anelasticity in the mantle [*Varga,* 1998]:

$$\tan \delta_{lmf} = \frac{k_{lm}^I(f)}{k_{lm}^R(f)}. \tag{28}$$

Note that if $k_{lm}^I(f) = 0$, also $\tan \delta_{lmf} = 0$. It may be important to know these parameters. Indeed, the initial phases of some semisecular tidal constituents, like the 18.6-year tide, consist entirely of them and the astronomical longitudes of the Sun and the Moon. Indeed eq.(24) vanishes for the even $(l - 2p = 0)$ zonal $(m = 0)$ long period $(l - 2p + q = 0)$ perturbations. For a time span T_{obs} shorter than their periods they could be fitted and removed from the time series provided that their periods and initial phases are exactly known. The same remarks will hold for the ocean tides.

The quantities $H_l^m(f)$ appearing in eq.(19) are the coefficients of the harmonic expansion of the tide generating potential. They contain the lunisolar ephemerides information and define the modulus of the vertical shift V/g in the equipotential surface at the Earth's surface for $r = R_\oplus$ with respect to the case in which only proper Earth's gravity is considered; g is the acceleration of gravity at the equator. The values used in the present calculation are those recently released by *Roosbeek* [1996]. Their accuracy is of the order of 10^{-7} m. In order to use them in place of the coefficients of *Cartwright and Edden* [1973], which have a different normalization, a multiplication for a suitable conversion factor [*McCarthy,* 1996] has been performed.

The eq.(18) is the dynamical, non central part of the geopotential due to the response of the solid Earth to the forcing lunisolar tidal solicitation. In the linear Lagrange equations of perturbation theory for the rates of change of the Keplerian elements of a satellite it plays the role of the perturbative function $\mathcal{R}$. The latter is the non central part of the total mechanical energy of the satellite, with the sign reversed, according to the geophysical convention. The

Lagrange equations are [*Kaula*, 1966]:

$$\frac{da}{dt} = \frac{2}{na}\frac{\partial\mathcal{R}}{\partial\mathcal{M}}, \tag{29}$$

$$\frac{de}{dt} = \frac{1-e^2}{na^2e}\frac{\partial\mathcal{R}}{\partial\mathcal{M}} - \frac{(1-e^2)^{1/2}}{na^2e}\frac{\partial\mathcal{R}}{\partial\omega}, \tag{30}$$

$$\frac{di}{dt} = \cos i\frac{1}{na^2(1-e^2)^{1/2}\sin i}\frac{\partial\mathcal{R}}{\partial\omega} -$$

$$-\frac{1}{na^2(1-e^2)^{1/2}\sin i}\frac{\partial\mathcal{R}}{\partial\Omega}, \tag{31}$$

$$\frac{d\Omega}{dt} = \frac{1}{na^2(1-e^2)^{1/2}\sin i}\frac{\partial\mathcal{R}}{\partial i}, \tag{32}$$

$$\frac{d\omega}{dt} = -\cos i\frac{1}{na^2(1-e^2)^{1/2}\sin i}\frac{\partial\mathcal{R}}{\partial i} +$$

$$+\frac{(1-e^2)^{1/2}}{na^2e}\frac{\partial\mathcal{R}}{\partial e}, \tag{33}$$

$$\frac{d\mathcal{M}}{dt} = n - \frac{1-e^2}{na^2e}\frac{\partial\mathcal{R}}{\partial e} - \frac{2}{na}\frac{\partial\mathcal{R}}{\partial a}. \tag{34}$$

In it $n = (GM_\oplus)^{1/2}a^{-3/2} = 2\pi/P$ is the mean motion of the satellite. It is an observed fact that the secular motions are the dominant perturbation in the elements ω, Ω, $\mathcal{M}$ of geodetically useful satellites. So, taking as constants a, e, i and consider as linearly variable in time $\omega, \Omega, \mathcal{M}$ and θ, besides σt, the following expressions, at first order, may be worked out:

$$\Delta\Omega_f = \frac{g}{na^2\sqrt{1-e^2}\sin i}\sum_{l=0}^{\infty}\sum_{m=0}^{l}(\frac{R}{r})^{l+1}\times$$

$$\times A_{lm}\sum_{p=0}^{l}\sum_{q=-\infty}^{+\infty}\frac{dF_{lmp}}{di}G_{lpq}\frac{1}{f_p}k_{lm}^{(0)}H_l^m\sin\gamma_{flmpq}, \tag{35}$$

$$\Delta\omega_f = \frac{g}{na^2\sqrt{1-e^2}}\sum_{l=0}^{\infty}\sum_{m=0}^{l}(\frac{R}{r})^{l+1}\times$$

$$\times A_{lm}\sum_{p=0}^{l}\sum_{q=-\infty}^{+\infty}[\frac{1-e^2}{e}F_{lmp}\frac{dG_{lpq}}{de} - \frac{\cos i}{\sin i}\frac{dF_{lmp}}{di}G_{lpq}]\frac{1}{f_p}k_{lm}^{(0)}H_l^m\sin\gamma_{flmpq}, \tag{36}$$

where:

$$\gamma_{flmpq} = (l-2p)\omega + (l-2p+q)\mathcal{M} + m(\Omega-\theta) + \sigma t - \delta_{lmf}, \tag{37}$$

$$f_p = (l-2p)\dot\omega + (l-2p+q)\dot{\mathcal{M}} + m(\dot\Omega - \dot\theta) + \sigma. \tag{38}$$

Only the perturbations whose periods are much longer than those of the orbital satellites motions, which, typically, amount to a few hours, are relevant for many geophysical and astronomical applications. This implies that in the summations of eqs.(35)-(36) only those terms in which the rate of the mean anomaly does not appear must be retained, i. e. the condition:

$$l - 2p + q = 0 \tag{39}$$

must be fulfilled. Moreover, if the effect of Earth's diurnal rotation, which could introduce periodicities of the order of 24 hours, is to be neglected, one must retain only those terms in which the non negative multiplicative coefficient j_1 of the Greenwich sidereal time in σt coincides to the order m of the tidal constituent considered: in this way in f_p the contributions of $\dot{\theta}$ are equal and opposite, and cancel out. With these bounds on l, m, p and q the circular frequencies of the perturbations of interest become:

$$f_p = \dot{\Gamma}_f + (l - 2p)\dot{\omega} + m\dot{\Omega} \tag{40}$$

in which:

$$\dot{\Gamma}_f = (j_2 - m)\dot{s} + j_3\dot{h} + j_4\dot{p} + j_5\dot{N'} + j_6\dot{p}_s. \tag{41}$$

In the performed calculation only the degree $l = 2$ constituents have been considered due to the smallness of the $k_{3m}^{(0)}$ and $H_3^m(f)$. For $l = 2$, p runs from 0 to 2, and so, in virtue of the condition $l - 2p + q = 0$, q assumes the values -2, 0, 2. From an inspection of the table of the eccentricity functions $G_{lpq}(e)$ in [*Kaula*, 1966] it turns out that $G_{20-2} = G_{222} = 0$, while $G_{210} = (1 - e^2)^{-3/2}$. For this combination of l, p and q the relation $l - 2p = 0$ is fulfilled: the frequencies of the perturbations are, in this case, given by:

$$f_p = \dot{\Gamma}_f + m\dot{\Omega}. \tag{42}$$

In Tab.1, Tab.2 and Tab.3 the results for the nodal lines of LAGEOS and LAGEOS II, and the apsidal line of LAGEOS II are shown; since the observable quantity for ω is $ea\dot{\omega}$ [*Ciufolini*, 1996], the calculation for the perigee of LAGEOS, due to the notable smallness of the eccentricity of its orbit, have not been performed. The tidal lines for which the analysis was performed have been chosen also in order to make a comparison with the perturbative

amplitudes of the ocean tides based on the results of EGM96 gravity model which will be shown in the next Section.

The results presented in Tab.1, Tab.2 and Tab.3 can be compared to those of *Dow* [1988] and Carpino [*Bertotti and Carpino*, 1989]. In doing so it must be kept in mind that both these authors have neglected not only the contribution of k_{lm}^+ but also the anelasticity and the frequency dependence of the Love numbers $k_{lm}^{(0)}$. Moreover, Dow includes in his analysis, in a not entirely clear manner, also the indirect influences of the oblateness of the Earth [*Balmino,* 1974; *Dow,* 1988; *Casotto*, 1989]. Obviously, in their analyses the LAGEOS II is not present since it was launched only in 1992. Another important factor to be considered is the actual sensitivity in measurements of Ω and ω, in the sense that the eventual discrepancies between the present results and the other ones must be not smaller than the experimental error in the Keplerian elements if one wishes to check the theoretical assumptions behind the different models adopted. Carpino has analyzed the inclination and the node of LAGEOS only. His value for the zonal 18.6-year tide is -1087.24 mas, while Tab.1 gives -1079.38 mas; the difference amounts to 7.86 mas, the 0.72 % of the "elastic", frequency-independent Carpino's value. Considering that for the node Ω of LAGEOS the present accuracy is of the order of the mas, 7.86 mas could be in principle detected, allowing for a discrimination between the different models adopted in the calculation. For the K_1 tide, one of the most powerful constraints in perturbing the satellites' orbits, Tab.1 quotes 1744.38 mas against 2144.46 mas of Carpino's result; the gap is 400.08 mas, the 18.6 % of Carpino. In the sectorial band, the present analysis quotes for the K_2 -92.37 mas while Carpino has -97.54 mas; there is a difference of 5.17 mas, the 5.3 % of the Carpino's value. It must pointed out that there are other tidal lines for which the difference falls below the mas level, as is the case for the 9.3-year tide. As it could be expected, the major differences between the present "anelastic", frequency-dependent calculations and the other ones based on a single, real value for the Love number k_2 lie in the diurnal band: in it the contribution of anelasticity is not particularly relevant, but, as already pointed out, the elastic part of $k_{21}^{(0)}$ is strongly dependent on frequencies of the tidal spectrum. May be interesting to note that when the calculation have been repeated with the same value $k_2 = 0.317$ adopted by Carpino, his results have been obtained again.

Up to now the effects induced by the Earth's flattening and the Earth's rotation have been

neglected. If they have to be analyzed, it is necessary to take in the free space potential's expansion the part [*Wahr,* 1981b]:

$$Re\ g \sum_{l=2}^{\infty} \sum_{m=0}^{l} \sum_{f} H_l^m(f)(\frac{R}{r})^{l+3} k_{lm}^+(f)\ Y_{l+2}^m(\phi,\ \lambda) \tag{43}$$

and work out it in the same manner as done for the spherical nonrotating Earth's contribution. In applying the standard transformation to the orbital elements of [*Kaula,* 1966] one has to substitute everywhere l with $l+2$. So the equations for the perturbations on the node and the perigee become:

$$\Delta\Omega_f^+ = \frac{g}{na^2\sqrt{1-e^2}\sin i} \sum_{l=0}^{\infty} \sum_{m=0}^{l} (\frac{R}{r})^{l+3} A_{l+2\ m} H_l^m \times$$
$$\times k_{lm}^+ \frac{1}{f_p} \sum_{p=0}^{l} \sum_{q=-\infty}^{+\infty} \frac{dF_{l+2\ mp}}{di} G_{l+2\ pq} \sin\gamma_{fl+2\ mpq}, \tag{44}$$

$$\Delta\omega_f^+ = \frac{g}{na^2\sqrt{1-e^2}} \sum_{l=0}^{\infty} \sum_{m=0}^{l} (\frac{R}{r})^{l+3} A_{l+2\ m} k_{lm}^+ H_l^m \frac{1}{f_p} \times$$
$$\times \sum_{p=0}^{l} \sum_{q=-\infty}^{+\infty} [\frac{1-e^2}{e} F_{l+2\ mp} \frac{dG_{l+2\ pq}}{de} -$$
$$- \frac{\cos i}{\sin i} \frac{dF_{l+2\ mp}}{di} G_{l+2\ pq}] \sin\gamma_{fl+2\ mpq}. \tag{45}$$

The corrections induced by the Earth's flattening and rotation, due to the smallness of k_{lm}^+, have been calculated only for those tidal lines which have resulted to be the most powerful in perturbing the node and the perigee of LAGEOS and LAGEOS II, i.e. the zonal 18.6-year tide and the K_1. Adopting the values quoted in [*Dehant et al.,* 1999] for k_{lm}^+ and the eq.(44) and eq.(45) it is possible to obtain the results summarized in Tab.4. The quoted values, with the exception of the node of LAGEOS II, fall below the mas level resulting, at the present, undetectable.

Concerning the mismodeling of the solid tidal perturbations of the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II, particularly important for the gravitomagnetic experiment, by inspecting the analytical expressions of their perturbations given in eqs.(35)-(36) it is possible to note that almost all quantities entering in them are very well determined with the possible exception of the inclination i and the Love number k. Concerning the in-clination i, by assuming $\delta i = 0.5$ mas [*Ciufolini,* 1989], we have calculated $\left| \frac{\partial A(\Omega)}{\partial i} \right| \delta i$ and

$\left|\frac{\partial A(\omega)}{\partial i}\right|\delta i$ for the 18.6-year tide, K_1, and S_2 which are the most powerful tidal constituents in perturbing LAGEOS and LAGEOS II orbits, as can be inferred from Tab.1-Tab.3. The results are of the order of 10^{-6} mas, so that we can neglect the effect of uncertainties in the inclination determination. About the Love numbers k_2, we have assessed the uncertainties in them by calculating for certain tidal constituents the factor $\delta k_2/k_2$; k_2 is the average on the values released by the most reliable models and δk_2 is its standard deviation. According to the recommendations of the Working Group of Theoretical Tidal Model of the Earth Tide Commission (http://www.astro.oma.be/D1/EARTH_TIDES/wgtide.html), in the diurnal band we have chosen the values released by *Mathews et al.* [1995], *McCarthy* [1996] and the two sets by *Dehant et al.* [1999]. For the zonal and sectorial bands we have included also the results of *Wang* [1994]. The uncertainties calculated in the Love numbers k_2 span from 0.4% to 1.5% for the tides of interest.

3   Ocean tidal perturbations on the nodal and apsidal lines of LAGEOS and LAGEOS II

The effects of the ocean tides [*Defant,* 1961; *Neumann et al.,* 1966; *Pekeris and Akkad,* 1969; *Hendershott and Munk,* 1970; *Hendershott,* 1972; 1973; *Schwiderski,* 1980; 1981; *Gill,* 1982, *Zahel,* 1997], for a constituent of given frequency f, can globally accounted for by means of the potential:

$$U_f = \sum_{l=0}^{\infty}\sum_{m=0}^{l}\sum_{+}^{-}(\frac{R_\oplus}{r})^{l+1}A_{lmf}^{\pm}\sum_{p=0}^{l}\sum_{q=-\infty}^{+\infty}F_{lmp}(i)G_{lpq}(e)\begin{bmatrix}\cos\gamma_{flmpq}^{\pm}\\\sin\gamma_{flmpq}^{\pm}\end{bmatrix}_{l-m\ odd}^{l-m\ even}, \qquad (46)$$

in which:

$$\gamma_{flmpq}^{\pm} = (l-2p)\omega + (l-2p+q)\mathcal{M} + m(\Omega-\theta) \pm (\sigma t - \varepsilon_{lmf}^{\pm}), \qquad (47)$$

$$A_{lmf}^{\pm} = 4\pi G R_\oplus \rho_w \frac{(1+k_{lf}^{'})}{2l+1}C_{lmf}^{\pm}. \qquad (48)$$

In eq.(46) expressions like $\sum_+^- A^{\pm}\cos{(a\pm b)}$ mean $A^+\cos{(a+b)} + A^-\cos{(a-b)}$. The sign $+$ refers to the progressive (westwards) waves while the sign $-$ is for the waves moving from West to East. Moreover, ρ_w denotes the water density [*McCarthy,* 1996], $k_l^{'}$ are the load Love numbers [*Farrell,* 1972] accounting for the ocean loading [*Jentzsch,* 1997], the $C_{lmf}^{\pm}$ are the

ocean tidal coefficients of EGM96 gravity model [*Lemoine et. al.*, 1998] and $\varepsilon_{lmf}^{\pm}$ are the phase shifts due to hydrodynamics of the oceans.

The equations for the tidal ocean perturbations may be worked out as already done for the solid Earth tides in the previous Section. At first order, one obtains:

$$\Delta\Omega_f = \frac{1}{na^2\sqrt{1-e^2}\sin i}\sum_{l=0}^{\infty}\sum_{m=0}^{l}\sum_{+}^{-}(\frac{R}{r})^{l+1}A_{lmf}^{\pm}\times$$

$$\times\sum_{p=0}^{l}\sum_{q=-\infty}^{+\infty}\frac{dF_{lmp}}{di}G_{lpq}\frac{1}{f_p}\left[\begin{array}{c}\sin\gamma_{flmpq}^{\pm}\\-\cos\gamma_{flmpq}^{\pm}\end{array}\right]_{l-m\ odd}^{l-m\ even}, \tag{49}$$

$$\Delta\omega_f = \frac{1}{na^2\sqrt{1-e^2}}\sum_{l=0}^{\infty}\sum_{m=0}^{l}\sum_{+}^{-}(\frac{R}{r})^{l+1}A_{lmf}^{\pm}\times$$

$$\times\sum_{p=0}^{l}\sum_{q=-\infty}^{+\infty}[\frac{1-e^2}{e}F_{lmp}\frac{dG_{lpq}}{de}-\frac{\cos i}{\sin i}\frac{dF_{lmp}}{di}G_{lpq}]\frac{1}{f_p}\left[\begin{array}{c}\sin\gamma_{flmpq}^{\pm}\\-\cos\gamma_{flmpq}^{\pm}\end{array}\right]_{l-m\ odd}^{l-m\ even}, \tag{50}$$

in which:

$$f_p = (l-2p)\dot\omega + (l-2p+q)\dot{\mathcal{M}} + m(\dot\Omega-\dot\theta)\pm\sigma. \tag{51}$$

It should be noted that the frequencies of the perturbations given by the eq.(51) are different, in general, from the frequencies of the solid Earth tidal perturbations given by eq.(38). While for the solid tides the diurnal modulation due to $\dot\theta$ cancels out automatically if one considers those terms in which $j_1 = m$, for the ocean tides, in general, this does not happen because of the presence of the Eastwards waves due to the non equilibrium pattern of the ocean tidal bulge. Long periodicities can be obtained considering those combinations of l, p and q for which $l-2p+q = 0$ and retaining the Westward prograde terms with $j_1 = m$. Only in this way in eq.(51) the contributions of $\dot\theta$ are equal and opposite, and cancel out. With these bounds on l, m, p and q the frequencies of the perturbations of interest become:

$$f_p = \dot\Gamma_f + (l-2p)\dot\omega + m\dot\Omega. \tag{52}$$

It is worthwhile noting that the frequencies f_p are identical to those of solid tidal perturbations, so that the satellites cannot distinguish one effect from the other. For $l = 2$, as for the solid tides, $l-2p = 0$ holds. The frequencies of the perturbations are, in this case, given by:

$$f_p = \dot\Gamma_f + m\dot\Omega. \tag{53}$$

If l is odd the situation is different because now all the terms with p running from 0 to l must be considered and q is different from zero: $q = 2p - l$. So, the rates of the perturbations include the contribute of $\dot{\omega}$.

The eqs.(49)-(50) have been adopted in order to compute the amplitudes of the ocean tidal perturbations on the Keplerian elements Ω, ω, $\mathcal{M}$ and i for LAGEOS and LAGEOS II. The calculation have been performed, considering only the progressive waves, for the following tidal lines:

M_m (065.455), S_a (056.554), M_f (075.555), S_{sa} (057.555);

K_1 (165.555), O_1 (145.555), P_1 (163.555), Q_1 (135.655);

K_2 (275.555), M_2 (255.555), S_2 (273.555), N_2 (245.655), T_2 (272.556).

For each of these tidal constituents the following terms have been calculated:

$l = 2$, $p = 1$, $q = 0$ because the eccentricity functions $G_{lpq}(e)$ for $p = 0$, $q = -2$ and $p = 2$, $q = 2$ vanish.

$l = 3$, $p = 1$, $q = -1$ and $l = 3$, $p = 2$, $q = 1$ because G_{30-3} and G_{333} are not quoted in (Kaula 1966) due to their smallness: indeed, the $G_{lpq}(e)$ are proportional to $e^{|q|}$.

$l = 4$, $p = 2$, $q = 0$ because the other admissible combinations of l, p and q give rise to negligible eccentricity functions. So, also for $l = 4$ the condition $l - 2p = 0$, in practice, holds and the constituents of degree $l = 2$ and $l = 4$ generate detectable perturbations with identical periods. Similar analyses can be found in [*Christodoulidis*, 1978; *Dow*, 1988]. When Christodoulidis performed his study, which is relative to only five constituents, neither the LAGEOS nor the LAGEOS II were in orbit, while Dow has sampled the tidal spectrum for LAGEOS in a poorer manner with respect to this study in the sense that, for each constituent, only the terms of degree $l = 2$ have been considered with the exception of the K_1 whose $l = 4$ contribution has been also analyzed. Moreover, when these work have been realized there were a few coefficients C_{lmf}^{+} available with relevant associated errors.

It should be pointed out that the values obtained for the coefficients C_{lmf}^{+} are, in general, biased by the effects of the anelasticity of the solid Earth's mantle and by all other phenomena which have not been explicitly modeled in the nominal background models used [*Lemoine et al.*, 1998]. For example, in the values recovered for the S_a are included climatological effects which have not gravitational origin; in the S_2 coefficients are also included the variations of the

atmospheric pressure due to the atmospheric tides.

In Tab.6, Tab.7 and Tab.8 the present results for the nodes Ω of LAGEOS and LAGEOS II, and the argument of perigee ω for LAGEOS II are quoted.

From an accurate inspection of Tab.6, Tab.7 it is possible to note that, for the nodes Ω, only the even degree terms give an appreciable contribute; the $l = 3$ part of the spectrum is totally negligible. This so because $\Delta\Omega_f$ is proportional to the $G_{lpq}(e)$ functions which, in turn, are, in general, proportional to $e^{|q|}$: in this case the eccentricity functions used are $G(e)_{31-1} = e(1 - e^2)^{-5/2} = G(e)_{321}$.

Among the long period zonal tides, the Solar annual tide S_a (056.554) exerts the most relevant action on the nodes, with an associated percent error in the amplitudes of 6.7 %. It is interesting to compare for this tidal line the ocean tidal perturbations of degree $l = 2$ $A^{ocean}_{LAGEOS}(\Omega) = -20.55$ mas, $A^{ocean}_{LAGEOSII}(\Omega) = 37.7$ mas, with those due to the solid Earth tides $A^{solid}_{LAGEOS}(\Omega) = 9.96$ mas, $A^{solid}_{LAGEOSII}(\Omega) = -18.28$ mas. The ocean amplitudes amount to 204 % of the solid ones, while for the other zonal constituents they vary from 9.9 % for the M_m to the 18.5 % of the S_{sa}. This seems to point toward that the recovered value of C^+_{20f} for the S_a is biased by other climatological effects than the tides; indeed, for all the other tidal lines, zonal or not, the ocean tidal perturbations of degree $l = 2$ amount to almost 10 % of the corresponding solid Earth tides perturbations.

The terms of degree $l = 2$ of the tesseral tides K_1 (165.555) and P_1 (163.555) induce very large perturbations on the node of LAGEOS and, to a lesser extent, of LAGEOS II; Tab.6 and Tab.7 quote 156.55 mas, -11.49 mas for the former and -35.69 mas, -3 mas for the latter. The associated percent errors are 3.8 % and 8.1 %, respectively, for the two constituents. For the terms of degree $l = 4$, which have the same periods of those of degree $l = 2$, the situation is analogous: Tab.6 and Tab.7 quote 4.63 mas and -0.32 mas for LAGEOS and 41.58 and 0.7 mas for LAGEOS II. The percent errors associated with K_1 and P_1, for $l = 4$, are 3.9 % and 8 % respectively. For all the tesseral lines investigated the ocean tidal perturbations of degree $l = 2$ are, in general, the 10 % of the solid tidal perturbations.

Among the sectorial tides, the most relevant in perturbing the nodes of LAGEOS satellites, on large temporal scales, is the S_2 (273.555): Tab.6 and Tab.7 quote 9.45 mas ($l = 2$) and 15.08 mas ($l = 4$) for LAGEOS, and 6.87 mas ($l = 2$) and 6.79 mas ($l = 4$) for LAGEOS II. The

associated percent errors are 3.9 % for $l = 2$ and 5.2 % for $l = 4$. The S_2 ocean perturbation of degree $l = 2$ amounts to nearly 5 % of the corresponding solid tide.

About the zonal 18.6-year and 9.3-year tides, recently both Starlette and LAGEOS SLR satellites passed their 19th year in orbit and this span of time is now adequate to get reliable information about these tides [*Cheng*, 1995; *Eanes*, 1995] due to their slow frequencies they can be adequately modeled in terms of the equilibrium theory through the H_l^m coefficients and a complex Love number accounting for the anelasticity of the mantle. So, concerning them the results quoted for the solid tides can be considered adequately representative.

In Tab.8 the amplitudes of the perturbations on the argument of perigee ω for LAGEOS II are quoted. For this orbital element the factor:

$$\frac{1 - e^2}{e} F_{lmp} \frac{dG_{lpq}}{de} - \frac{\cos i}{\sin i} \frac{dF_{lmp}}{di} G_{lpq} \tag{54}$$

to which $\Delta\omega_f$ is proportional makes the contributions of the $l = 3$ terms not negligible. For the even degree terms the situation is quite similar to that of Ω in the sense that the most influent tidal lines are the S_a, K_1, P_1 and S_2.

Once again, among the long period tides the S_a exhibits a characteristic behavior. Indeed, its $l = 3$ contributions are much stronger than those of the other zonal tides. This fact could be connected to the large values obtained in its C_{30f}^+ coefficient and people believe that it partially represents north to south hemisphere mass transport effects with an annual cycle nontidal in origin. The $l = 3$ terms present, in general, for the perigee of LAGEOS II a very interesting spectrum also for the tesseral and sectorial bands: for LAGEOS II there are lots of tidal lines which induce, on large temporal scales, very relevant perturbations on ω, with periods of the order of an year or more. Above all, it must be quoted the effect of K_1 line for $p = 1$, $q = -1$: the perturbation induced amounts to -1136 mas with period of -1851.9 days. These values are comparable to the effects induced by the solid Earth tidal constituent of degree $l = 2$ on the node Ω of LAGEOS. By comparing the $l = 2$ terms with the corresponding solid tides, it can be noted that also for the perigee the proportions are the same already seen for the nodes.

About the mismodeling of the ocean tidal orbital perturbations, which are the worst determined part of the spectrum of the Earth tidal response, the major source of uncertainties in their amplitudes resides in the EGM96 coefficients C_{lmf}^+ and the ocean loading parameters

k'_l. Concerning the ocean loading, *Pagiatakis* [1990] in a first step has recalculated k'_l for an elastic, isotropic and non-rotating Earth: for $l < 800$ he claims that his estimates differ from those by *Farrell* [1972], calculated with the same hypotheses, of less than 1%. Subsequently, he added to the equations, one at a time, the effects of anisotropy, rotation and dissipation; for low values of l their effects on the results of the calculations amount to less than 1%. By inspecting Tab.6-Tab.8 it has been decided to calculate $\left| \frac{\partial A(\omega)}{\partial k'_l} \right| \delta k'_l$ of the perigee of LAGEOS II for K_1 $l = 3$ $p = 1$. First, we have calculated mean and standard deviation of the values for k'_3 by Farrell and Pagiatakis obtaining $\delta k'_3/k'_3 = 0.9\%$, in according to the estimates by Pagiatakis. Then, by assuming in a pessimistic way that the global effect of the departures from these symmetric models yield to a total $\delta k'_3/k'_3 = 2\%$, we have obtained $\delta \omega^{II} = 5.5$ mas. Subsequently, for this constituent and for all other tidal lines we have calculated the effect of the mismodeling of C^+_{lmf} as quoted in EGM96.

4 The influence of the tides on the detection of the Lense-Thirring drag

The theoretical general relativistic expressions for the gravitomagnetic precessions of the node and the perigee of a test particle in the field of a massive rotating central body -the Lense-Thirring effect- are given by:

$$\dot{\Omega}_{LT} = \frac{G}{c^2} \frac{2J}{a^3(1-e^2)^{3/2}}, \tag{55}$$

$$\dot{\omega}_{LT} = -\frac{G}{c^2} \frac{2J}{a^3(1-e^2)^{3/2}} 3\cos i, \tag{56}$$

in which:

J angular momentum of the central rotating body, g $cm^2 s^{-1}$. For the Earth its value is $5.9 \cdot 10^{40}$ g $cm^2 s^{-1}$.

c speed of light in vacuo, cm s^{-1}.

According to [*Ciufolini*, 1996; *Ciufolini et al.* 1997; 1998], it should be possible to detect the Lense-Thirring shift at a 20 % level of accuracy through the formula:

$$\delta\dot{\Omega}^I + c_1\delta\dot{\Omega}^{II} + c_2\delta\dot{\omega}^{II} = 60.05 \times \mu_{LT}, \tag{57}$$

where $c_1 \simeq 0.295$, $c_2 \simeq -0.35$, μ_{LT} is an adimensional scale parameter which is 1 in general relativity and 0 in Newtonian mechanics and $\delta\Omega^I$, $\delta\Omega^{II}$, $\delta\omega^{II}$ are the residuals, in mas, of the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II calculated, e.g., with the aid of the NASA-Goddard software GEODYN II. In the analyses performed with the real data [*Ciufolini et al.*, 1997; 1998] the Lense-Thirring parameter μ has been purposely left out the dynamical models of GEODYN II by setting $\mu = 0$ so that the residuals can entirely account for it. General relativity predicts for eq.(57) a secular trend with a slope of 60.05 mas/y if it is calculated by means of eqs.(55)-(56). Eq.(57) has been recently investigated in [*Ciufolini et al.*, 1998] for a 4 years time span.

In view of a refinement of the error budget of this experiment the results obtained in Sec. 2 and Sec. 3 have been used in order to calculate the mismodeled amplitudes of the solid and ocean tidal perturbations $\delta\Omega^I$, $\delta\Omega^{II}$ and $\delta\omega^{II}$; they have been subsequently compared with the gravitomagnetic precessions over 4 years $\Delta\Omega^I_{LT} = 124$ mas, $\Delta\Omega^{II}_{LT} = 126$ mas and $\Delta\omega^{II}_{LT} = -228$ mas. In Tab.5 we have quoted those solid tidal lines whose mismodeled perturbative amplitudes amount to 1%, at least, of the gravitomagnetic shifts. It turns out that only the 18.6-year tide and K_1 exceed this cutoff. Regarding the ocean tides, it turns out from Tab.9 that the perigee of LAGEOS II is more sensitive to the mismodeling of the ocean part of the Earth response to the tide generating potential. In particular, the effect of K_1 $l = 3$ $p = 1$ $q = -1$ is relevant with a total $\delta\omega = \left| \frac{\partial A(\omega)}{\partial C^+_{lmf}} \right| \delta C^+_{lmf} + \left| \frac{\partial A(\omega)}{\partial k'_l} \right| \delta k'_l$ of 64.5 mas amounting to 28.3 % of $\Delta\omega_{LT}$ over 4 years.

The combination y_{LT} is useful since it should vanish if calculated for the even zonal contributions C_{20} and C_{40} of the geopotential [*Ciufolini*, 1996]. More precisely, the right side of eq.(57) should become equal to zero if the left side were calculated for any of these two even zonal contributions, both of static and dynamical origin; the nearer to zero is the right side, the smaller is the systematic uncertainty in μ due to the contribution considered.

In order to test preliminarily this important feature for the case of tides, in a very conservative way the results obtained in the present work have been used in eq.(57) assuming, for the sake of clarity and in order to make easier the comparison with [*Ciufolini et al.*, 1997], a 1 year time span and the nominal values of the calculated tidal perturbative amplitudes, as if the zonal solid and ocean tides were not at all included in the GEODYN II dynamical models

so that the residuals should account entirely for them. The results are released in Tab.10 and Tab.11. The values of $\delta\mu$ quoted there for the various zonal tidal lines may be considered as the systematic error in μ due to the chosen constituents, if considered one at a time by neglecting any possible reciprocal correlation among the other tidal lines. Tab.10 and Tab.11 show that the percent error in the general relativistic value of μ due to the 18.6-year tide, the most dangerous one in recovering the LT since it superimposes to the gravitomagnetic trend over time spans of a few year, amounts to 21.9 %, while for all the other zonal tides it reduces to 0.1 % or less. This means that, even if fully present in the combined residuals, the $l = 2$ $m = 0$ tides, with the exception of the 18.6-year tide, do not affect the recovery of μ_{LT} by means of y_{LT} so that there is no need including them into the final least-squares scheme. It is interesting to compare the present results to those released in [*Ciufolini et al.* 1997] for the 18.6-year tide. The value -0.219 due to the solid component for μ quoted in Tab.10 must be compared to -0.361 in [*Ciufolini et al.* 1997], with an improvement of 39.3 %. In the cited work there is no reference to any estimate of the mismodeling of the 18.6-year tide, so that we have used the nominal tidal perturbative amplitudes released in it: $A(\Omega^I) = -997$ mas, $A(\Omega^{II}) = 1805$ mas and $A(\omega^{II}) = -1265$ mas. These figures for the perturbative amplitudes due to the solid Earth tide of 18.6-year are notably different from those quoted in the present study. In [*Ciufolini et al.*, 1997] the theoretical framework in which those numbers have been calculated (F. Vespe, private communication, 1999) is based on the assumption of a spherical, static, elastic Earth with a single nominal value of $k_2 = 0.317$ used for the entire tidal spectrum. The inclusion of the tiny corrections due to the Earth's flattening and rotation on the perturbative amplitudes of Ω and ω for the 18.6-year tide could allow to slightly improve the related uncertainty in μ; it would amount to 20.6 %. But since the present-day accuracy in laser ranging measurements could hardly allow to detect these small effects, their utilization in eq.(57) is debatable.

Remember that the result quoted for the 18.6-year tide is obtained in the worst possible case, i.e. a time span of only 1 year and the assumption that the residuals have been built up by neglecting completely the zonal tides in the dynamical models. Recall that if the residuals accounted for the 18.6-year tide it would not be possible to view it as an empirically fit quantity unless a T_{obs} of, at least, 18.6 years is adopted. However, if, more realistically, we calculate eq.(57) with the mismodeled amplitudes quoted in the first row of Tab.5 for the 18.6-year tide

we obtain $\delta\mu =\leq -3.51 \cdot 10^{-3}$. This conclusion has been confirmed also by calculating the time average of the 18.6-year tide over different time spans by using the mismodeling level adopted here [*Pavlis and Iorio*, 2001].

Even though a cancellation is not expected as for the first two even zonal constituents, calculating the left hand side of eq.(57) for the other tides yields, at least, an order of magnitude of their effect on μ. An interesting, unpredicted feature stands out for the odd zonal ocean tides. The contribution of $l = 3$ zonal ocean tidal nominal perturbations over 1 year to $\delta\mu$ can be found in Tab.12 and Tab.13. From an inspection of them it is clear that the sensitivity of perigee of LAGEOS II to the $l = 3$ part of the ocean tidal spectrum may affect the recovery of the Lense-Thirring parameter μ by means of the proposed combined residuals, especially as far as S_a and S_{sa} are concerned. This fact agrees with the results of Tab.9 which tells us that the mismodeled parts of S_a and S_{sa} are not negligible fractions of $\Delta\omega_{LT}^{II}$. However, if the mismodeled amplitudes are employed in eq.(57) it can be seen that, over 1 year, a cancellation of the order of 10^{-1} (S_a $l = 3$ $p = 2$) and 10^{-2} (S_a $l = 3$ $p = 1$; S_{sa} $l = 3$ $p = 1, 2$) takes place. The contributions of the mismodeling on M_m and M_f are completely negligible. So, we can conclude that also the $l = 3$ part of the zonal ocean tidal spectrum may not affect the combined residuals in a sensible manner if the $l = 3$ part of S_a and S_{sa} is properly accounted for.

5   Conclusions

The detection of the general relativistic Lense-Thirring drag in the field of the Earth by means of the combined residuals of the two LAGEOS laser-ranged satellites is affected, among other factors, also by the Earth solid and ocean tidal perturbations.

Concerning the solid tides of degree $l = 2$, the most effective constituents turn out to be the semisecular 18.6-year, the K_1 and the S_2 which induce on the examined orbital elements perturbations of the order of $10^2 - 10^3$ mas with periods ranging from 111.24 days for the S_2 on LAGEOS II to 6798.38 days for the 18.6-year tide.

Regarding the ocean tides, we have analyzed the degree $l = 2, 3, 4$ terms of 13 constituents by using the data of the EGM 96 model. If, from one hand, the nominal ocean amplitudes of degree $l = 2$ amount to almost 10% of the solid ones for the same degree, from the other hand

the obtained results for the perigee of LAGEOS II show that this orbital element is very sensitive to the degree $l = 3$ part of the tidal spectrum. Indeed, for many tidal lines the perturbations on it with $l = 3$ $p = 1, 2$ $q = -1, 1$ exhibit long periods of some years and amplitudes of the order of magnitude of those due to the solid Earth tides for the node. The K_1 $l = 3$ $p = 1$ $q = -1$ line stands out with its period of -1851.9 days (5.07 years) and amplitude of 1136 mas. Such term has a 5.2 % error.

The calculations performed here have been used, in view of a refinement of the error budget of the gravitomagnetic LAGEOS experiment, in order to check preliminarily which tidal constituents are really important in perturbing the combined residuals so to fit and remove them from the data, if possible, or, at least, to evaluate the systematic error induced by them. Tab. 5 and Tab. 9 show that, over a 4 years time span the nodes of the two LAGEOS are sensitive to the even components of the 18.6-year line, the K_1 and the S_a at a 1% level at least. Moreover, the perigee of LAGEOS II turns out to be very sensitive to the $l = 3$ part of the ocean tidal spectrum.

Concerning the very long period tides, an accurate calculation from first principles of their amplitudes, periods and initial phases turns out to be important in the case of an observational time span shorter than their periods. Indeed, while for a biasing true linear trend one can only assess as more accurately as possible the error induced by it, in the case of the arc of sinusoid of a harmonic perturbation whose period is longer than the adopted time span it should be possible to fit and remove it from the signal, without affecting the trend of interest, provided that its period and initial phases are exactly known. In any case it could be possible to assess reliably its role in the error budget.

However, in the Lense-Thirring experiment we have shown in a preliminary way that this should not be necessary, at least for the semisecular even zonal 18.6-year tide independently of the time series length. Indeed, if calculated at the level of accuracy shown in this paper, it affects the combined residuals at a level $\leq 10^{-3}$. The other even zonal tides do not create problems. Also the $l = 3$ $m = 0$ tides, and this is an unpredicted feature cancel out at a level $\leq 10^{-1}$-10^{-2} if they are properly accounted for in building up the residuals. The results presented in this paper not only confirm the usefulness of the formula by Ciufolini in canceling out the $l = 2, 4$ $m = 0$ tides, but also extend its validity to the $l = 3$ $m = 0$ part of the tidal

response spectrum.

The impact of the tesseral ($m = 1$) and sectorial ($m = 2$) tidal constituents on the combined residuals, which are fully sensitive to them, over different time spans and time steps will be the subject of a forthcoming numerical analysis based on the results presented here.

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Solid Earth tidal perturbations on the node Ω of LAGEOS					
Tide	l=2, p=1, q=0				
	Period (days)	Amplitude (mas)	k_2 Love number	$H_l^m(f)$ (meters)	$\tan\delta_{lmf}$
055.565	6798.38	-1079.38	0.315	0.02792	-0.01715
055.575	3399.19	5.23	0.313	0.000272	-0.015584
056.554 S_a	365.27	9.96	0.307	-0.00492	-0.01135
057.555 S_{sa}	182.62	31.21	0.305	-0.03099	-0.01029
065.455 M_m	27.55	5.28	0.302	-0.03518	-0.00782
075.555 M_f	13.66	4.94	0.301	-0.06659	-0.007059
165.545	1232.94	-41.15	0.259	-0.007295	-0.00554
165.555 K_1	1043.67	1744.38	0.257	0.3687012	-0.0055933
165.565	904.77	203.02	0.254	0.050028	-0.005653
163.555 P_1	-221.35	136.44	0.286	-0.12198	-0.005017
145.555 O_1	-13.84	19	0.297	-0.26214	-0.00484
135.655 Q_1	-9.21	2.42	0.297	-0.05019	-0.00483
274.556	-1217.55	1.68	0.301	0.000625	-0.00431
274.554	-1216.73	-6.63	0.301	-0.00246	-0.00431
275.555 K_2	521.835	-92.37	0.301	0.0799155	-0.00431
273.555 S_2	-280.93	182.96	0.301	0.2940	-0.00431
272.556 T_2	-158.80	6.04	0.301	0.0171884	-0.00431
255.555 M_2	-14.02	19.63	0.301	0.6319	-0.00431
245.655 N_2	-9.29	2.49	0.301	0.12099	-0.00431

Table 1: Perturbative amplitudes on the node Ω of LAGEOS due to solid Earth tides. In the first column the Doodson number of each constituent is quoted followed by the Darwin's name, when it is present. The tidal lines are listed in order of decreasing periods. The coefficients $H_l^m(f)$ are those recently calculated by Roosbeek and multiplied by suitable normalization factors (IERS standards) in order to make possible a comparison with those of Cartwright and Edden. $\tan\delta_{lmf}$ expresses the phase lag of the solid Earth response with respect to the tidal potential due to the anelasticity in the mantle.

Solid Earth tidal perturbations on the node Ω of LAGEOS II					
Tide	l=2, p=1, q=0				
	Period (days)	Amplitude (mas)	k_2 Love number	$H_l^m(f)$ (meters)	$\tan\delta_{lmf}$
055.565	6798.38	1982.16	0.315	0.02792	-0.01715
055.575	3399.19	-9.61	0.313	-0.000272	-0.015584
056.554 S_a	365.27	-18.28	0.307	-0.00492	-0.01135
057.555 S_{sa}	182.62	-57.31	0.305	-0.03099	-0.01029
065.455 M_m	27.55	-9.71	0.302	-0.03518	-0.00782
075.555 M_f	13.66	-9,08	0.301	-0.06659	-0.007059
165.565	-621.22	-58.31	0.254	0.050028	-0.005653
165.555 K_1	-569.21	-398	0.257	0.3687012	-0.0055933
165.545	-525.23	7.33	0.259	-0.007295	-0.005541
163.555 P_1	-138.26	35.65	0.286	-0.1219	-0.005017
145.555 O_1	-13.33	7.66	0.297	-0.26214	-0.00484
135.655 Q_1	-8.98	0.98	0.297	-0.05019	-0.00483
275.555 K_2	-284.6	-92.51	0.301	0.079915	-0.004318
274.556	-159.96	-0.40	0.301	0.000625	-0.00431
274.554	-159.95	1.6	0.301	-0.00246	-0.0043
273.555 S_2	-111.24	-133.04	0.301	0.29402	-0.00431
272.556 T_2	-85.27	-5.96	0.301	0.017188	-0.00431
255.555 M_2	-13.03	-33.05	0.301	0.6319	-0.004318
245.655 N_2	-8.84	-4.35	0.301	0.12099	-0.00431

Table 2: Perturbative amplitudes on the node Ω of LAGEOS II due to solid Earth tides. In the first column the Doodson number of each constituent is quoted followed by the Darwin's name, when it is present. The tidal lines are listed in order of decreasing periods. The coefficients $H_l^m(f)$ are those recently calculated by Roosbeek and multiplied by suitable normalization factors (IERS standards) in order to make possible a comparison with those of Cartwright and Edden. $\tan\delta_{lmf}$ expresses the phase lag of the solid Earth response with respect to the tidal potential due to the anelasticity in the mantle.

Solid Earth tidal perturbations on the perigee ω of LAGEOS II					
Tide	l=2, p=1, q=0				
	Period (days)	Amplitude (mas)	k_2 Love number	$H_l^m(f)$ (meters)	$\tan\delta_{lmf}$
055.565	6798.38	-1375.58	0.315	0.02792	-0.01715
055.575	3399.19	6.66	0.313	-0.000272	-0.015584
056.554 S_a	365.27	12.69	0.307	-0.00492	-0.01135
057.555 S_{sa}	182.62	39.77	0.305	-0.03099	-0.01029
065.455 M_m	27.55	6.74	0.302	-0.03518	-0.00782
075.555 M_f	13.66	6.30	0.301	-0.06659	-0.007059
165.565	-621.22	290.43	0.254	0.050028	-0.005653
165.555 K_1	-569.21	1982.14	0.257	0.3687012	-0.0055933
165.545	-525.23	-36.52	0.259	-0.007295	-0.005541
163.555 P_1	-138.26	-177.56	0.286	-0.1219	-0.005017
145.555 O_1	-13.33	-38.16	0.297	-0.26214	-0.00484
135.655 Q_1	-8.98	-4.92	0.297	-0.05019	-0.00483
275.555 K_2	-284.6	-88.19	0.301	0.079915	-0.004318
274.556	-159.96	-0.38	0.301	0.000625	-0.00431
274.554	-159.95	1.52	0.301	-0.00246	-0.0043
273.555 S_2	-111.24	-126.83	0.301	0.29402	-0.00431
272.556 T_2	-85.27	-5.68	0.301	0.017188	-0.00431
255.555 M_2	-13.03	-31.9	0.301	0.6319	-0.004318
245.655 N_2	-8.84	-4.15	0.301	0.12099	-0.00431

Table 3: Perturbative amplitudes on the perigee ω of LAGEOS II due to solid Earth tides. In the first column the Doodson number of each constituent is quoted followed by the Darwin's name, when it is present. The tidal lines are listed in order of decreasing periods. The coefficients $H_l^m(f)$ are those recently calculated by Roosbeek and multiplied by suitable normalization factors (IERS standards) in order to make possible a comparison with those of Cartwright and Edden. $\tan\delta_{lmf}$ expresses the phase lag of the solid Earth response with respect to the tidal potential due to the anelasticity in the mantle.

Corrections due to Earth's flattening and rotation to solid tidal perturbations				
Tide	l=2, p=2, q=0			
	$\Delta\Omega_{LAGEOS}$ (mas)	$\Delta\Omega_{LAGEOSII}$ (mas)	$\Delta\omega_{LAGEOSII}$ (mas)	k_{2m}^+
055.565	0.43	1.28	0.19	-0.00094
165.555 K_1	-0.97	-9.49	0.85	-0.00074

Table 4: Corrections to the Earth solid tidal perturbations on the nodes of LAGEOS and the perigee of LAGEOS II due to the Earth's flattening and Earth's rotation. The values adopted for k_{2m}^+ are those quoted by *Dehant* [1999].

Mismodeled solid tidal perturbations on nodes of LAGEOS and LAGEOS II and perigee of LAGEOS II							
Tide	$\Delta\Omega_{LT}^I = 124$ mas $\Delta\Omega_{LT}^{II} = 126$ mas $\Delta\omega_{LT}^{II} = -228$ mas P_{obs}=4 years						
	$\delta k_2/k_2$ (%)	$\delta\Omega^I$ (mas)	$\delta\Omega^I/\Delta\Omega_{LT}^I$ (%)	$\delta\Omega^{II}$ (mas)	$\delta\Omega^{II}/\Delta\Omega_{LT}^{II}$ (%)	$\delta\omega^{II}$ (mas)	$\delta\omega^{II}/\Delta\omega_{LT}^{II}$ (%)
055.565	1.5	-16.5	13.3	30.3	24	-21	9.2
165.555 K_1	0.5	9	7.2	-2	1.6	10.2	4.4

Table 5: Mismodeled solid tidal perturbations on nodes Ω of LAGEOS and LAGEOS II and the perigee ω of LAGEOS II compared to their gravitomagnetic precessions over 4 years.

Ocean tidal perturbations on the node Ω of LAGEOS											
Tide	$l=2,\ p=1,\ q=0$			$l=3,\ p=1,\ q=-1$			$l=3,\ p=2,\ q=1$			$l=4,\ p=2,\ q=0$		
	P	A	E	P	A	E	P	A	E	P	A	E
065.455 Mm	27.55	-0.54	14.4	28	-10^{-4}	66.6	27.11	10^{-4}	66.6	-	-	-
056.554 Sa	365.27	-20.55	6.7	464.67	-10^{-2}	10	300.91	10^{-2}	10	-	-	-
075.555 Mf	13.66	-0.62	7.8	13.77	-10^{-4}	112	13.55	10^{-4}	112	-	-	-
057.555 Ssa	182.62	-5.98	9.4	204.5	-10^{-3}	27.2	164.9	10^{-3}	27.2	-	-	-
165.555 K1	1043.67	156.55	3.8	2684.2	-0.36	5.2	647.76	10^{-2}	5.2	1043.67	4.63	3.9
163.555 P1	-221.35	-11.49	8.1	-195.95	10^{-3}	18.5	-254.3	-10^{-3}	18.5	-221.35	-0.32	8
145.555 O1	-13.84	-2	2.9	-13.72	10^{-3}	3.2	-13.95	10^{-4}	3.2	-13.84	10^{-2}	5.7
135.655 Q1	-9.21	-0.28	13.5	-9.16	10^{-4}	25	-9.26	10^{-5}	25	-9.21	-10^{-3}	20
275.555 K2	521.83	-6.24	11.1	751.5	-10^{-2}	5.5	399.7	10^{-2}	5.5	521.83	-9.58	15.4
273.555 S2	-280.93	9.45	3.9	-241.24	10^{-3}	7.1	-336.25	-10^{-2}	7.1	-280.93	15.08	5.2
272.556 T2	-158.8	0.28	75	-145.3	10^{-4}	50	-175	-10^{-3}	50	-158.8	0.44	100
255.555 M2	-14.02	2.03	0.9	-13.9	-10^{-4}	7.4	-14.14	10^{-3}	7.4	-14.02	2.08	2.8
245.655 N2	-9.29	0.3	4.6	-9.2	10^{-5}	12.5	-9.3	10^{-4}	12.5	-9.29	0.3	8.3

Table 6: Perturbative amplitudes on the node Ω of LAGEOS due to ocean tides. P indicates the periods in days, A the amplitudes in mas and E the percent error in the C_{lmf}^{+}. The values employed for them and the related errors are those quoted in EGM96 model.

Ocean tidal perturbations on the node Ω of LAGEOS II											
Tide	$l=2,\ p=1,\ q=0$			$l=3,\ p=1,\ q=-1$			$l=3,\ p=2,\ q=1$			$l=4,\ p=2,\ q=0$		
	P	A	E	P	A	E	P	A	E	P	A	E
065.455 Mm	27.55	1	14.4	26.65	10^{-3}	66.6	28.5	-10^{-3}	66.6	-	-	-
056.554 Sa	365.27	37.71	6.7	252.8	0.13	10	657.5	-0.35	10	-	-	-
075.555 Mf	13.6	1.13	7.8	13.43	10^{-4}	112	13.89	-10^{-4}	112	-	-	-
057.555 Ssa	182.62	10.98	9.4	149.41	10^{-2}	27.2	234.8	-10^{-2}	27.2	-	-	-
165.555 K1	-569.21	-35.69	3.8	-1851.9	-1.02	5.2	-336.3	-10^{-3}	5.2	-569.21	41.58	3.9
163.555 P1	-138.26	-3	8.1	-166.23	-10^{-2}	18.5	-118.35	-10^{-4}	18.5	-138.26	3.29	8
145.555 O1	-13.3	-0.8	2.9	-13.5	-10^{-2}	3.2	-13.12	-10^{-4}	3.2	-13.3	0.7	5.7
135.655 Q1	-8.98	-0.11	13.5	-9.08	-10^{-3}	25	-8.89	10^{-5}	25	-8.98	0.11	20
275.555 K2	-284.6	-6.24	11.1	-435.38	-0.13	5.5	-211.4	10^{-2}	5.5	-284.6	-5.95	15.4
273.555 S2	-111.2	-6.87	3.9	-128.6	-10^{-2}	7.1	-97.9	10^{-2}	7.1	-111.2	-6.79	5.2
272.556 T2	-85.27	-0.277	75	-95.14	-10^{-3}	50	-77.25	10^{-3}	50	-85.27	-0.274	100
255.555 M2	-13.03	-3.46	0.9	-13.2	-10^{-3}	7.4	-12.83	10^{-3}	7.4	-13.03	-2.2	2.8
245.655 N2	-8.8	-0.46	4.6	-8.9	-10^{-3}	12.5	-8.7	10^{-4}	12.5	-8.8	-0.34	8.3

Table 7: Perturbative amplitudes on the node Ω of LAGEOS II due to ocean tides. P indicates the periods in days, A the amplitudes in mas and E the percent error in the C_{lmf}^{+}. The values employed for them and the related errors are those quoted in EGM96 model.

Ocean tidal perturbations on the perigee ω of LAGEOS II											
Tide	$l=2,\ p=1,\ q=0$			$l=3,\ p=1,\ q=-1$			$l=3,\ p=2,\ q=1$			$l=4,\ p=2,\ q=0$		
	P	A	E	P	A	E	P	A	E	P	A	E
065.455 Mm	27.55	-0.69	14.4	26.65	-1.53	66.6	28.5	1.64	66.6	-	-	-
056.554 Sa	365.27	-26.17	6.7	252.8	-114.35	10	657.55	297.34	10	-	-	-
075.555 Mf	13.66	-0.78	7.8	13.43	-0.58	112	13.89	0.60	112	-	-	-
057.555 Ssa	182.6	-7.62	9.4	149.41	-22.95	27.2	234.8	36.07	27.2	-	-	-
165.555 K1	-569.21	177.76	3.8	-1851.9	-1136	5.2	-336.28	346.6	5.2	-569.21	-3.95	3.9
163.555 P1	-138.26	14.95	8.1	-166.2	-28.97	18.5	-118.35	34.67	18.5	-138.2	-0.31	8
145.555 O1	-13.3	4	2.9	-13.55	-13.7	3.2	-13.12	22.3	3.2	-13.3	-10^{-2}	5.7
135.655 Q1	-8.98	0.58	13.5	-9.08	-1.17	25	-8.89	1.92	25	-8.98	-10^{-2}	20
275.555 K2	-284.6	-5.95	11.1	-435.3	214.23	5.5	-211.4	87.3	5.5	-284.6	-2.49	15.3
273.555 S2	-111.2	-6.55	3.9	-128.6	98.47	7.1	-97.9	62.9	7.1	-111.2	-2.85	5.2
272.556 T2	-85.2	-0.26	75	-95.1	5.2	50	-77.2	3.54	50	-85.2	-0.11	100
255.555 M2	-13.03	-3.3	0.9	-13.2	9.7	7.4	-12.83	7.9	7.4	-13.03	-0.92	2.8
245.655 N2	-8.48	-0.44	4.6	-8.94	1.95	12.5	-8.75	1.6	12.5	-8.84	-0.14	8.3

Table 8: Perturbative amplitudes on the perigee ω of LAGEOS II due to ocean tides. P indicates the periods in days, A the amplitudes in mas and E the percent error in the C_{lmf}^{+}. The values employed for them and the related errors are those quoted in EGM96 model.

Tide	δC$^+$/C$^+$ (%)	δΩ^I (mas)	δΩ^I/ΔΩ$^I_{LT}$ (%)	δΩ^{II} (mas)	δΩ^{II}/ΔΩ$^{II}_{LT}$ (%)	δω^{II} (mas)	δω^{II}/Δω$^{II}_{LT}$ (%)
Mismodeled ocean tidal perturbations on nodes of LAGEOS and LAGEOS II and perigee of LAGEOS II							
ΔΩ$^I_{LT}$ = 124 mas ΔΩ$^{II}_{LT}$ = 126 mas Δω$^{II}_{LT}$ = −228 mas P$_{obs}$ = 4 years							
S_a l=2 p=1 q=0	6.7	1.37	1.1	2.5	1.9	-	-
S_a l=3 p=1 q=-1	10	-	-	-	-	11.4	5
S_a l=3 p=2 q=1	10	-	-	-	-	29.7	13
S_{sa} l=3 p=1 q=-1	27.2	-	-	-	-	6.2	2.7
S_{sa} l=3 p=2 q=1	27.2	-	-	-	-	9.8	4.3
K_1 l=2 p=1 q=0	3.8	5.9	4.7	1.3	1	6.75	2.9
K_1 l=3 p=1 q=-1	5.2	-	-	-	-	64.5	28.3
K_1 l=3 p=2 q=1	5.2	-	-	-	-	18	7.9
K_1 l=4 p=2 q=0	3.9	-	-	1.6	1.2	-	-
P_1 l=3 p=1 q=-1	18.5	-	-	-	-	5.3	2.3
P_1 l=3 p=2 q=1	18.5	-	-	-	-	6.4	2.8
K_2 l=3 p=1 q=-1	5.5	-	-	-	-	11.7	5
K_2 l=3 p=2 q=1	5.5	-	-	-	-	4.8	2
S_2 l=3 p=1 q=-1	7.1	-	-	-	-	6.9	3
S_2 l=3 p=2 q=1	7.1	-	-	-	-	4.4	1.9
T_2 l=3 p=1 q=-1	50	-	-	-	-	2.6	1.1

Table 9: Mismodeled ocean tidal perturbations on nodes Ω of LAGEOS and LAGEOS II and the perigee ω of LAGEOS II compared to their gravitomagnetic precessions over 4 years. The effect of the ocean loading has been neglected. When the 1% cutoff has not been reached a - has been inserted. The values quoted for K_1 $l = 3$ $p = 1$ includes also the mismodeling in the ocean loading coefficient k_3' assumed equal to 2%.

Tide	A(Ω$_I$) (mas)	A(Ω$_{II}$) (mas)	A(ω$_{II}$) (mas)	δμ
Solid Earth even zonal tidal contributions to δμ				
l=2, m=0, p=1, q=0				
055.565	-1079.38	1982.16	-1375.58	-0.219
055.575	5.23	-9.61	6.66	$1.06 \cdot 10^{-3}$
056.554 S_a	9.95	-18.28	12.69	$2.02 \cdot 10^{-3}$
057.555 S_{sa}	31.21	-57.31	39.77	$6.33 \cdot 10^{-3}$
065.455 M_m	5.28	-9.71	6.74	$1.07 \cdot 10^{-3}$
075.555 M_f	4.94	-9.08	6.3	$1 \cdot 10^{-3}$

Table 10: Contribution of the even zonal solid tidal constituents to δμ by means of the formula $\delta\dot\Omega^I + \delta\dot\Omega^{II} \times 0.295 - \delta\dot\omega^{II} \times 0.35 = 60.05 \times \mu$.

Tide	A(Ω$_I$) (mas)	A(Ω$_{II}$) (mas)	A(ω$_{II}$) (mas)	δμ
Ocean even zonal tidal contributions to δμ				
l=2, m=0, p=1, q=0				
056.554 Sa	-20.55	37.71	-26.17	$-3.68 \cdot 10^{-3}$
057.555 Ssa	-5.98	10.98	-7.62	$-1.28 \cdot 10^{-3}$
065.455 Mm	-0.54	1	-0.69	$-8.78 \cdot 10^{-5}$
075.555 Mf	-0.62	1.13	-0.78	$-1.73 \cdot 10^{-4}$

Table 11: Contribution of the even zonal ocean tidal constituents to δμ by means of the formula $\delta\dot\Omega^I + \delta\dot\Omega^{II} \times 0.295 - \delta\dot\omega^{II} \times 0.35 = 60.05 \times \mu$.

Ocean odd zonal tidal contributions to $\delta\mu$				
Tide	l=3, m=0, p=1, q=-1			
	$A(\Omega_I)$ (mas)	$A(\Omega_{II})$ (mas)	$A(\omega_{II})$ (mas)	$\delta\mu$
056.554 Sa	-0.063	0.13	-114.35	0.66
057.555 Ssa	$-9 \cdot 10^{-3}$	0.028	-22.95	0.133
065.455 Mm	$-4 \cdot 10^{-4}$	$1 \cdot 10^{-3}$	-1.53	$-8.93 \cdot 10^{-3}$
075.555 Mf	$-1 \cdot 10^{-4}$	$7 \cdot 10^{-4}$	-0.58	$3.41 \cdot 10^{-3}$

Table 12: Contribution of the odd zonal ocean tidal constituents to $\delta\mu$ by means of the formula $\delta\dot{\Omega}_I + \delta\dot{\Omega}^{II} \times 0.295 - \delta\dot{\omega}^{II} \times 0.35 = 60.05 \times \mu$ for $p = 1,\ q = -1$.

Ocean odd zonal tidal contributions to $\delta\mu$				
Tide	l=3, m=0, p=2, q=1			
	$A(\Omega_I)$ (mas)	$A(\Omega_{II})$ (mas)	$A(\omega_{II})$ (mas)	$\delta\mu$
056.554 Sa	0.047	-0.36	297.34	-1.72
057.555 Ssa	$7 \cdot 10^{-3}$	-0.044	36.07	-0.209
065.455 Mm	$4 \cdot 10^{-4}$	$-2 \cdot 10^{-3}$	1.642	$-9.55 \cdot 10^{-3}$
075.555 Mf	$1.79 \cdot 10^{-4}$	$-7 \cdot 10^{-4}$	-0.60	$-3.52 \cdot 10^{-3}$

Table 13: Contribution of the odd zonal ocean tidal constituents to $\delta\mu$ by means of the formula $\delta\dot{\Omega}^I + \delta\dot{\Omega}^{II} \times 0.295 - \delta\dot{\omega}^{II} \times 0.35 = 60.05 \times \mu$ for $p = 2,\ q = 1$.